A1 Discounting and present worth factors

A1.1 Introduction

Introduction

This appendix provides tables of present worth factors for use in discounting. A discussion on the economic principles of discounted cash flow and present worth is contained in chapter 2 of this manual.

In this appendix

This appendix contains the following topics:

A1.2 Discounting

Discounting

Benefits and costs generally arise throughout the life of projects and to calculate their present worth or present value (PV) they need to be discounted back to time zero. Based on a discount rate of 10 percent, sets of present worth factors have been calculated to convert future benefits and costs to their PVs (see table A1.1 and A1.2).

Some benefits and costs occur at a single point in time in which case single payment present worth factors (SPPWF) shall be used to discount the amounts to their PV. Other benefits and costs occur continuously over a number of years in which case either uniform series (USPWF) or arithmetic growth present worth factors (AGPWF) shall be used to discount the amounts to a PV, depending on whether the amounts are uniform or increase arithmetically over time (eg traffic and patronage growth).

When discounting accident benefits the traffic growth rate will need to be adjusted in accordance with the procedures in appendix A6 to determine the appropriate arithmetic growth rate to apply. External impacts are assumed to remain constant so the uniform present worth series should be used to obtain the PV of monetised impacts.

When discounting benefits or costs determined from a transportation model, the present worth factors specified in this appendix shall be used. If necessary, adjust values to time zero equivalents. Traffic growth rate may require a similar adjustment to time zero.

A1.3 Single payment present worth factor

Single payment present worth factor (SPPWF)

Where a single benefit or cost arises at some future time, a single payment present worth factor (SPPWF) shall be applied to calculate its PV.

The formula for determining SPPWF factors is:

$$SPPWF^{i}_{n} = \frac{1}{(1 + i)^{n}} = \frac{1}{1.10^{n}} \quad \text{for a 10\% discount rate}$$

where:

n = time in years after time zero, and

i = is the discount rate in percent.

The PV of a single benefit or cost at time n shall be calculated as follows:

PV of benefit (or cost) = $SPPWF^{i}_{n}$ x benefit (or cost)

Example 1

For a section of road resealed 15 years after time zero at a cost of $50,000, the PV of the reseal cost using a discount rate of 10% is:

PV = $50,000 x $SPPWF^{10}_{15}$

= $50,000 x 0.2394

= $11,970

A1.3 Single payment present worth factor, continued

Example 2

A project costing $2 million with a implementation period of 15 months starting in the 8th month after time zero, has the following cash flow for expenditure:

2nd half of year 1

Month	7	8	9	10	11	12	Total
$ (000's)	0	50	50	50	100	150	400

1st half of year 2

Month	13	14	15	16	17	18	Total
$ (000's)	200	200	300	300	200	100	1300

2nd half of year 2

Month	19	20	21	22	23	24	Total
$ (000's)	50	50	100	100	0	0	300

The PV of the implementation expenditure is:

Using annual SPPWF from table A1.1

PV = ($400,000 + $1,300,000) x $SPPWF^{10}_{1}$ + $300,000 x $SPPWF^{10}_{2}$

= $1,700,000 x 0.9091 + $300,000 x 0.8264

= $1,794,000

A more accurate calculation using quarterly SPPWF from table A1.2

PV = $150,000 x $SPPWF^{10}_{0.75}$ + $450,000 x $SPPWF^{10}_{1.00}$

+ $800,000 x $SPPWF^{10}_{1.25}$ + $350,000 x $SPPWF^{10}_{1.50}$

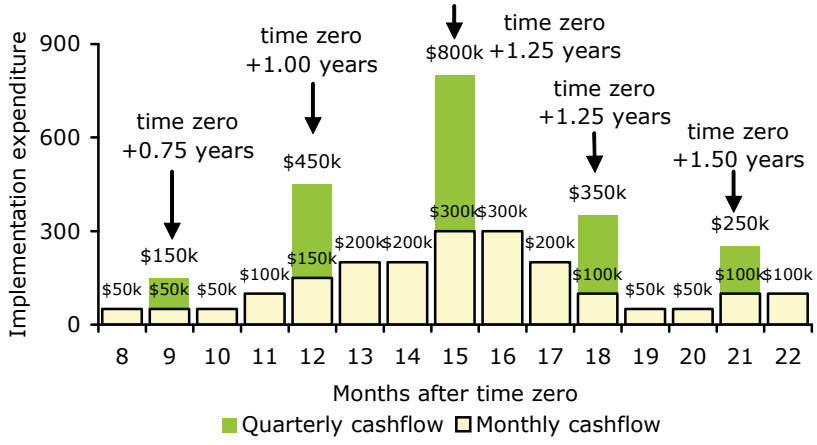
+ $250,000 x $SPPWF^{10}_{1.75}$

= $150,000 x 0.9310 + $450,000 x 0.9091

+ $800,000 x 0.8877 + $350,000 x 0.8668

+ $250,000 x 0.8464

= $1,774,000



A1.4 Uniform series present worth factor

Uniform series present worth factor (USPWF)

Where a series of equal benefits or costs arise each year or continuously over a period, uniform series present worth factors (USPWF) shall be applied to calculate their PV.

The USPWF factors shown in table A1.1 assume that the annual benefits or costs are evenly spread over each year and are continuously compounded.

The formula for determining these USPWF factors is:

$$USPWF^{i}_{n} = \frac{(1- (1 + i)^{-n})}{\log_e(1 + i)}$$

where:

n = time in years after time zero, and

i = is the discount rate in percent.

The PV of a time stream of equal annual benefits or costs shall be calculated as follows:

PV of benefits (or costs) = Annual benefit (or cost) x ($USPWF_e$- $USPWF_s$)

where:

s = the start year, and

e = the end year of the cost or benefit stream.

Example

If maintenance costs for the do minimum are $30,000 a year over a 27 year evaluation period (25 years plus 2 years to the start of construction), the PV of the maintenance costs is:

PV = \$30,000 x ($USPWF^{10}_{27}$ - $USPWF^{10}_{0}$)

= \$30,000 x (9.692 - 0)

= \$290,760

A1.5 Arithmetic growth present worth factor

Arithmetic growth present worth factor (AGPWF)

Where costs or benefits increase (or decrease) each year arithmetically, arithmetic growth present worth factors (AGPWF) together with the corresponding USPWF factors shall be applied to calculate their PVs. It shall be assumed that traffic growth is arithmetic.

The AGPWF factors shown in table A1.1 assume that the annual benefits or costs occur continuously throughout the year and are continuously compounded.

The formula for determining these AGPWF factors is:

$$AGPWF^{i}_{n} = [\log_e (1 + i)]^{-2} - n.(1 + i)^{-n}. [\log_e (1 + i)]^{-1} - (1 + i)^{-n}. [\log_e (1 + i)]^{-2}$$

where:

n = time in years after time zero, and

i = is the discount rate in percent.

The PV of a time stream of benefits or costs which increase or decrease arithmetically shall be calculated as follows:

$$\text{PV of benefits (or costs)} = \text{Annual benefits (or costs)} \times \{(USPWF_e - USPWF_s) + (R \times (AGPWF_e - AGPWF_s))\}$$

where:

R = the arithmetic growth rate at time zero,

s = the start year, and

e = the end year of the cost or benefit stream.

Example

If vehicle operating costs are $70,000 with traffic growth of 3 percent at time zero, and construction finishes 2 years from time zero, the PV of the vehicle operating costs on the new construction is:

$$PV = \$70{,}000 \times [(USPWF^{10}_{27} - USPWF^{10}_{2}) + 0.03 \times (AGPWF^{10}_{27} - AGPWF^{10}_{2})]$$

$$= \$70{,}000 \times [(9.692 - 1.821) + 0.03 \times (80.078 - 1.763)]$$

$$= \$715{,}430$$

A1.6 Annual present worth factors

Table A1.1 Annual present worth factors — for 10% discount rate

Time (years from time zero in period 1 July to 30 June)	Single payment SPPWF [1]	Time (years from time zero)	Uniform series USPWF [2]	Arithmetic growth AGPWF [2]
0	1.0000	0	0	0
1	0.9091	1	0.954	0.469
2	0.8264	2	1.821	1.763
3	0.7513	3	2.609	3.728
4	0.6830	4	3.326	6.230
5	0.6209	5	3.977	9.157
6	0.5645	6	4.570	12.409
7	0.5132	7	5.108	15.905
8	0.4665	8	5.597	19.572
9	0.4241	9	6.042	23.350
10	0.3855	10	6.447	27.190
11	0.3505	11	6.815	31.048
12	0.3186	12	7.149	34.890
13	0.2897	13	7.453	38.687
14	0.2633	14	7.729	42.415
15	0.2394	15	7.980	46.054
16	0.2176	16	8.209	49.592
17	0.1978	17	8.416	53.015
18	0.1799	18	8.605	56.316
19	0.1635	19	8.777	59.488
20	0.1486	20	8.932	62.529
21	0.1351	21	9.074	65.434
22	0.1228	22	9.203	68.204
23	0.1117	23	9.320	70.840
24	0.1015	24	9.427	73.342
25	0.0923	25	9.524	75.714
26	0.0839	26	9.612	77.958
27	0.0763	27	9.692	80.078
28	0.0693	28	9.765	82.078
29	0.0630	29	9.831	83.963
30	0.0573	30	9.891	85.736

[1] assuming cost or benefit occurs at end of year

[2] assuming costs or benefits for year occur continuously throughout the year and are continuously compounded.

A1.7 Quarterly present worth factors

Table A1.2 Quarterly single payment present worth factors — for 10% discount rate

Time (years from time zero in quarters from 1 July to 30 June)	SPPWF
0	1.0000
0.25	0.9765
0.50	0.9535
0.75	0.9310
1.00	0.9091
1.25	0.8877
1.50	0.8668
1.75	0.8464
2.00	0.8264
2.25	0.8070
2.50	0.7880
2.75	0.7694
3.00	0.7513
3.25	0.7336
3.50	0.7164
3.75	0.6995
4.00	0.6830
4.25	0.6669
4.50	0.6512
4.75	0.6359
5.00	0.6209